

## NARROWBAND LUMPED-ELEMENT MICROSTRIP FILTERS USING CAPACITIVELY-LOADED INDUCTORS

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### ABSTRACT

Coupling between microstrip resonators decreases very slowly as a function of the resonator separation. Therefore it is difficult to realize narrowband filters (e.g. <0.1% bandwidth) in microstrip form due to the very weak coupling values required. In this paper, we have developed a class of lumped-element filters that uses capacitively-loaded inductors to give frequency-dependent inductance values. A novel frequency-transformation technique is used in the design process. Using this approach, strong coupling can be used in narrowband filter designs. The frequency-dependent inductance transforms the filter to a narrower bandwidth than the original circuit prototype, and does not require hard-to-realize weak coupling. We present a 0.3% bandwidth superconducting microstrip prototype filter. It was designed with the coupling of a 1% bandwidth filter, and then transformed to 0.3% fractional bandwidth using an appropriate inductance slope parameter,  $\frac{dL}{d\omega}$ . Measurement showed good agreement with theory.

### INTRODUCTION

In narrowband microstrip filter designs, the requisite weak coupling is always a challenge. For example, it is hard to realize a very narrowband filter (e.g. < 0.1% bandwidth) in the convenient microstrip configuration using conventional coupling schemes due to the slow decay of the coupling as a function of the resonator element separation. To realize the weak coupling required by a narrowband filter, resonator elements in the filter have to be kept very far apart. This requires either a large circuit size or an elaborate package. In those cases, cavity type filters, which are usually quite large in size, or the stripline configuration, which is usually hard to package, often must be used. Those will inevitably increase the final system complexity and the engineering cost.

In this paper, a novel approach to solve this weak coupling problem in narrowband filter designs is introduced. Instead of using the conventional coupling approach, frequency-dependent inductors in lumped-element filter circuits are used. Design is accomplished by starting with a wideband conventional filter architecture and applying a frequency transformation to the circuit. This method results in very narrowband filters that other microstrip circuits can not realize, while still using strong coupling values in the design to keep the resonator sections close together in a compact form.

### PRINCIPLE OF THE FREQUENCY TRANSFORMATION TECHNIQUE

For simplicity, consider the lumped-element bandpass filter circuit in Fig. 1.

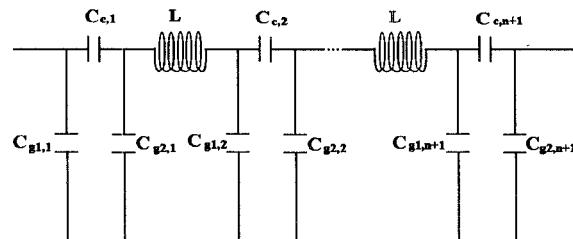


Figure 1. Topology of the lumped-element filter circuit

In this lumped-element filter circuit, all the inductors are transformed to the same inductance value  $L$ . In between adjacent inductors, a  $\pi$ -capacitor network is inserted. Similar  $\pi$ -capacitor networks are also used at

the input and output to match the appropriate circuit impedance. For an  $n$ -pole bandpass filter, there are  $n$  identical inductors and  $n+1$  different  $\pi$ -capacitor networks.

In each  $\pi$ -capacitor network, the series capacitor,  $C_{ci}$ , is the coupling capacitor between adjacent resonators. Narrowband filters always have very weak coupling between resonators, and require very small values of the coupling capacitors,  $C_{ci}$ . In microstrip realizations of this circuit, small  $C_{ci}$  values mean larger resonator separations, that are often impossible to realize due to the limited wafer size. In other cases, the filter performance will be very sensitive to the package dimensions.

In this paper, we describe a frequency transformation method to solve this weak coupling problem in the microstrip configuration. If a frequency-dependent inductor,  $L(\omega)$ , can be realized, we show that we can realize very narrowband filters without using weak coupling capacitances  $C_{ci}$ . The inductor slope parameter,  $\frac{dL}{d\omega}$ , will control the bandwidth of the filter,

while we can still use easy-to-realize strong coupling capacitors,  $C_{ci}$ , from the relatively wideband filter circuit prototype.

As can be shown, the real filter fractional bandwidth, after using the frequency transformation technique, will be

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{1 + \frac{\omega_0}{L} \frac{dL'(\omega)}{d\omega}} \Big|_{\omega_0} \frac{\Delta\omega_0}{\omega_0}$$

where  $\omega_0$  is the filter center frequency,  $\Delta\omega_0$  is the filter bandwidth in the circuit prototype, and  $L'(\omega)$  is the frequency-dependent inductance, with  $L'(\omega_0)=L$ . From this bandwidth transformation relationship, it can be seen that to achieve a narrowband filter, we can actually design a broader bandwidth circuit prototype filter with easily realizable coupling capacitors,  $C_{ci}$ , and then choose an appropriate inductor slope parameter,  $\frac{dL}{d\omega}$ , to achieve the narrowband filter performance.

It can also be shown that after the frequency transformation, the bandwidth of the filter is transformed to either narrower or broader bandwidths, depending on the positive or negative inductor slope parameter,  $\frac{dL}{d\omega}$ , while the filter response function,  $S_{21}$ , is

still conserved. This means that a Chebyshev filter design will still conserve the Chebyshev response after bandwidth transformation, a feature that is critical for

its applications in various situations of different bandwidths and center frequencies.

## AN EXAMPLE CIRCUIT USING THE FREQUENCY TRANSFORMATION TECHNIQUE

To demonstrate the above frequency transformation concept, consider the following example circuit. The specifications of the desired filter are: center frequency  $f_0=900$  MHz, 5 poles, fractional bandwidth  $w=0.28\%$ , passband ripple  $L=0.05$  dB.

If Chebyshev response is considered, this filter will require a weakest coupling of -51.1 dB, which is a level that is hard to reach in microstrip form due to poor isolation between resonators. The filter resonator elements would have to be placed very far apart to achieve this weak coupling level. (For an even narrower bandwidth such as 0.05%, the coupling required is only -66.1 dB. It is impossible to build this 0.05% filter using the conventional coupling scheme in microstrip configuration.)

If a similar filter is considered with exactly the same specifications except that the fractional bandwidth is now 1% instead of 0.28%, then this filter will require a weakest coupling of -40 dB, which is achievable in microstrip configuration. Starting with this 1% filter design using the topology shown in Figure 1, followed by the substitution of a frequency dependent inductor  $L'(\omega)$  in the designed circuit, we can easily get a new filter which has a bandwidth of 0.28%. The transmission and return loss response of the 1% filter is shown in curves *a* in Figure 2. Also shown in Figure 2, curves *b*, are the response of the filter after the frequency transformation, with a inductance value  $\frac{dL'(\omega)}{d\omega} \Big|_{\omega_0} = k = 9.085 \times 10^{-10} / \text{Hz}$ , and  $L'(\omega_0)=1.752 \times 10^{-8} \text{ H}$ .

From these response curves, we can clearly see that the Chebyshev approximation is still conserved, while the bandwidth of the filter is reduced from 1% to 0.28%, which is exactly the value calculated from the bandwidth transformation equation using the  $k$  and  $L$  values provided, through the frequency transformation. The realization of this frequency-dependent inductor will be discussed in the next section.

The deviation of the transmission responses between this 0.28% transformed filter in the  $\omega'$  domain and that of the original 1% filter in the  $\omega$  domain is calculated and plotted in Figure 3. Within the passband, the maximum deviation from the original Chebyshev function is less than 0.02 dB, while that outside of the

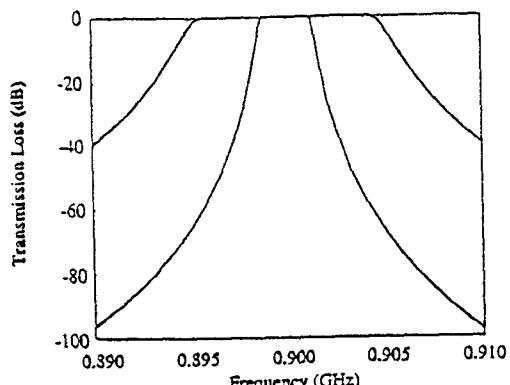


Figure 2a. Transmission loss response from the original 1% bandwidth filter and response of the 0.28% filter after frequency transformation.

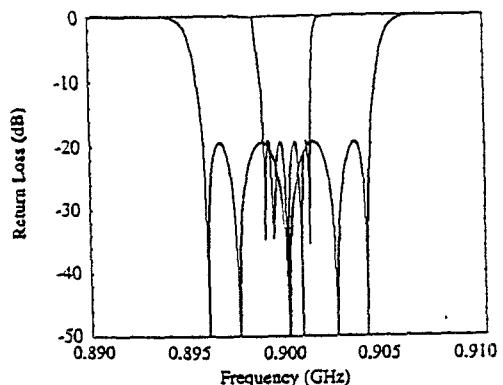


Figure 2b. Return loss response from the original 1% bandwidth filter and response of the 0.28% filter after frequency transformation.

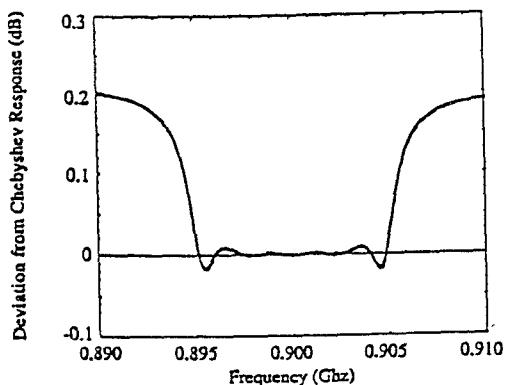


Figure 3. Deviation in dB from the Chebyshev responses between the 0.28% filter in the  $\omega'$  domain and that of 1% filter in the  $\omega$  domain.

passband is less than 0.2 dB at a 40 dB rejection frequency. These numbers show that the Chebyshev function is well preserved even after almost a 4-fold reduction in bandwidth.

## REALIZATION OF THE FREQUENCY-DEPENDENT INDUCTORS

The key for the frequency transformation technique described above is the slope of the inductor values as a function of frequency. In the usual transmission line realization of inductors, the inductor slope parameter  $k$  has a negative value because of the parasitic capacitance to ground. Thus, the bandwidth of the lumped-element filters designed using the conventional transmission line approximation of inductors will always have a broader bandwidth than the original design. (This phenomenon was reported by D. Swanson et al. in [1].) In order to achieve positive  $k$  values, which gives bandwidth transformation to the narrower side, other  $L(\omega)$  mechanisms have to be introduced into the circuit.

One simple realization of  $L(\omega)$  with a positive  $k$  is a single capacitor  $C$  in parallel with an inductor  $L_0$ . From the resultant impedance  $Z_{eq}$ ,

$$\frac{1}{Z_{eq}} = \frac{1}{j\omega L_0} + j\omega C$$

$$Z_{eq} = j\omega L'$$

we can calculate the equivalent inductance below

$$\omega_0 = \frac{1}{\sqrt{L_0 C}}$$

$$L' = \frac{L_0}{1 - \omega_0^2 L_0 C}$$

where  $L_0$  is the inductance of the inductor itself and  $C$  is the capacitance of the capacitor in parallel with the inductor. The slope parameter,

$$k = \frac{dL'(\omega)}{d\omega} \Big|_{\omega_0} = \frac{2\omega_0 L_0^2 C}{(1 - \omega_0^2 L_0 C)^2}$$

has a positive value. This parallel L-C component can be easily realized using a half-loop of inductor in parallel with an interdigital capacitor as in Figure 4.

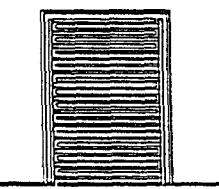


Figure 4. Layout of a microstrip realization of a frequency-dependent inductor.

## MEASUREMENT ON A 0.3% BANDWIDTH PROTOTYPE FILTER

In this section, we demonstrate the validity of the frequency transformation technique by experimentally constructing a narrowband filter. To demonstrate the concept, we built a 2-pole, 0.3% bandwidth bandpass filter at 900 MHz in microstrip form using superconducting niobium.

The prototype filter is built as follows: First, a relatively broadband filter circuit prototype (realizable coupling) is chosen with 1% bandwidth. The inductor and capacitor values are obtained for this 1% bandwidth filter. Second, for the desired bandwidth of 0.3%, the inductor slope parameter required is calculated from the bandwidth transformation equation

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{1 + \frac{\omega_0}{L} \frac{dL'(\omega)}{d\omega}} \Big|_{\omega_0} \frac{\Delta\omega_0}{\omega_0}$$

Third, the inductor slope parameter is realized from the equation  $L' = \frac{L_0}{1 - \omega^2 L_0 C}$  using the approach in the above

section, and the parallel capacitor value is obtained. Fourth, by realizing all the individual values of the inductors and capacitors, we will have a 0.3% bandwidth filter, while still using the more easily achievable stronger coupling from the 1% bandwidth filter circuit prototype. Figure 5 shows the layout of this 2-pole 0.3% bandwidth filter, and figure 6 shows the experimentally measured 0.3% bandwidth filter response, as well as the 1% bandwidth circuit prototype response and the 0.3% bandwidth simulation response from a full-wave electromagnetic simulator. This clearly demonstrates that the filter bandwidth is truly transformed from the 1% circuit prototype to the 0.3% desired bandwidth.

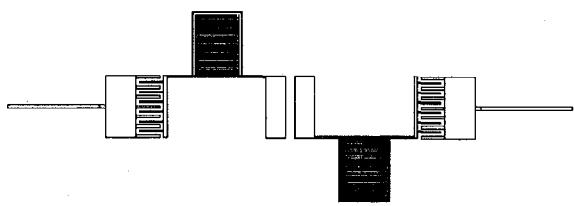


Figure 5. Layout of a 2-pole, 0.3% bandwidth filter realized on microstrip using the frequency transformation technique.

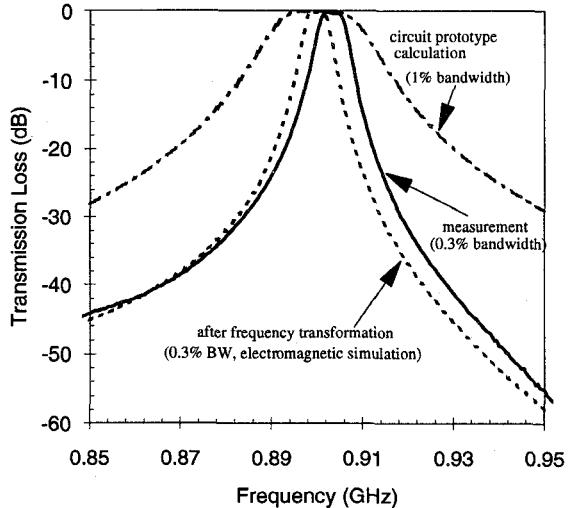


Figure 6. Measured 0.3% bandwidth filter response using a Nb superconducting microstrip structure, as well as that calculated for the 1% bandwidth filter circuit prototype and a full-wave electromagnetic simulation result.

## CONCLUSION

In conclusion, we have developed a new class of lumped-element filters that uses capacitively-loaded inductors to form frequency-dependent inductor values. A novel frequency transformation technique is applied to transform the filter bandwidth from the wideband circuit prototypes to narrowband filters. Using this technique, equivalent weak couplings, and therefore very narrowband filters, can be realized in a microstrip configuration. This frequency transformation method was also verified in a 0.3% bandwidth prototype filter design fabricated using niobium superconducting microstrip. Measurement showed good agreement with theory.

## ACKNOWLEDGMENT

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## REFERENCE

1. D. G. Swanson, Jr, R. Forse, and B. J. L. Nilsson, "A 10 GHz thin film lumped-element high temperature superconducting filter", 1992 IEEE MTT-S Digest, pp. 1191-1193.